ON THE CALCULATION OF ADIABATIC PROCESSES IN REAL GASES

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Means of calculating the final temperature of an isoentropic process, and also the temperature drops, are discussed. It is shown that in calculations of adiabatic processes it is advisable in some cases to use the proposed adiabatic exponent m.

In calculations of adiabatic processes it is first necessary, from the initial state and final pressure of the gas, to determine the final temperature of the isoentropic process.

If it is assumed that the parameters of a real gas obey the equation of a reversible adiabatic curve in its usual form

$$pv^k = \text{const}, \tag{1}$$

then [1]

$$k = -\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_s. \tag{2}$$

Using the equation of state of a real gas in the form

$$pv/RT = z, \tag{3}$$

we obtain the following expression for the adiabatic exponent,

$$k = \frac{c_p}{c_v}\,b, \tag{4}$$

in which

$$b = 1\Big/\left[1 - \frac{p}{z}\left(\frac{\partial z}{\partial p}\right)_T\right].$$

The adiabatic exponent k can be calculated for argon, air, carbon monoxide and dioxide, hydrogen, nitrogen, and oxygen using tabulated data [2]. The quantity $(\partial z/\partial p)_T$ is found either from the equation of state with $z = f(p, T)$, or by numerical differentiation from tables of compressibility factors.

For a linear dependence of z on pressure, with $T = \text{const}$, $b = z$.

The adiabatic exponent k can be determined from the velocity of sound in the gas [1].

By simultaneous solution of Eqs. (1) and (3) an expression is obtained for the final temperature of an isoentropic process,

$$T_2 = T_1 \frac{z_1}{z_2}\left(\frac{p_2}{p_1}\right)^{(k-1)/k}, \tag{5}$$

where $(k-1)/k$ is the arithmetic mean of the analogous exponents for the initial and final points.

The use of Eq. (5) has definite disadvantages, since it is necessary during the calculations to make use of two tables, namely compressibility factor and adiabatic exponent table.

As is well known [1],

$$ds = \frac{c_p dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp. \tag{6}$$

From Eq. (3) and the constancy of the entropy in a reversible adiabatic process,

$$\frac{dT}{T} = \frac{R}{c_p}\left[\left(\frac{\partial z}{\partial T}\right)_p T + z\right]\frac{dp}{p}. \tag{7}$$

Further, by introducing the notation

$$m = \frac{R}{c_p}\left[\left(\frac{\partial z}{\partial T}\right)_p T + z\right] \tag{8}$$

and taking the quantity $\bar{m}$ in the interval under consideration from p_1, T_1 to p_2, T_2 to be constant and equal to $(m_1 + m_2)/2$, we obtain, after integration of Eq. (7),

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\bar{m}}. \tag{9}$$

For z = 1 Eqs. (5) and (9) transform into the corresponding equations for an ideal gas. ·

The value of the partial derivative of the compressibility factor with respect to temperature is easily determined by numerical differentiation for equidistant points using values of the functions at these points [3].

For three interpolation points

$$\left(\frac{\partial z_0}{\partial T}\right)_p = \frac{1}{2\Delta T}(z_1 - z_{-1}). \tag{10}$$

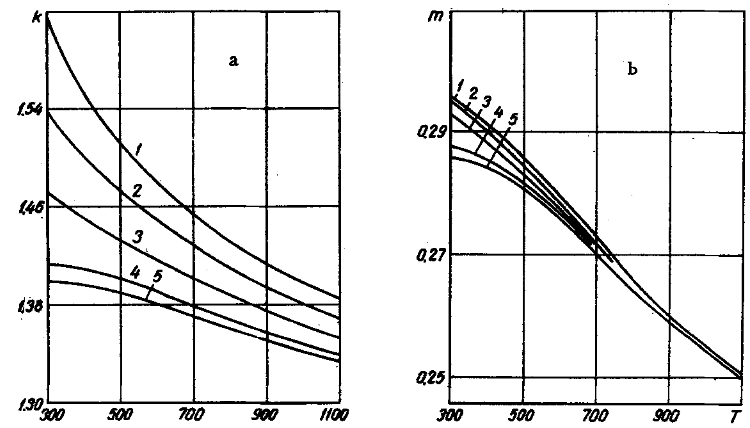
For five interpolation points

$$\left(\frac{\partial z_0}{\partial T}\right)_p = \frac{1}{12\Delta T}[8(z_1 - z_{-1}) - (z_2 - z_{-2})]. \tag{11}$$

For a large number of calculations even the accuracy of Eq. (10) is sufficient. On differentiating at the beginning or at the end of a table it is necessary to use the appropriate formulae [3]. Numerical differentiation is preferable to graphical both in the accuracy achievable and in the convenience of the calculation.

The heat capacity at constant pressure is usually tabulated. If not c_p can be found by numerical differentiation of the enthalpy using equations analogous to (10) and (11).

The figure shows the results of calculations of the adiabatic exponent.



The variation of the adiabatic exponents k(*a*) and m(b) for nitrogen with temperature and pressure: 1) for 10 Mn/m^2; 2) 7; 3) 4; 4) 1; 5) ideal gas state.

The error in determining the final temperature of an isoentropic process from Eqs. (5) and (9) depends on the errors in m and k themselves, the error in averaging them, and in Eq. (5) on the additional errors in z_1 and z_2.

An evaluation of the accuracy of the adiabatic exponents given in [2] showed that if use is made of the same initial data, i.e., p-v-T data and the thermodynamic properties of an ideal gas, then the errors in k and m are of the same order. However, the relative error in the exponent in Eq. (5) is 2 to 4 times greater than the relative error in the adiabatic exponent k:

$$\Delta\left(\frac{k-1}{k}\right)\Big/\frac{k-1}{k}=\frac{1}{k-1}\,\frac{\Delta k}{k}. \qquad (12)$$

Using the thermodynamic expression for $c_p - c_v$ [4] we obtain the relationship between m and k:

$$m=\sqrt{\left(\frac{c_p}{c_v}-1\right)Rz/c_pk}\,. \qquad (13)$$

Entropy tables can also be used to determine the final temperature of a reversible adiabatic process.

Entropies depend significantly on the pressure; one of the terms in its analytical formula is ln p, so that very extensive tables are necessary so that a linear interpolation can be made for intermediate values of the pressure. Interpolation of a much higher order requires a greater number of calculations. For example, from 1 to 4 Mn/m^2 on the 400° K isotherm for nitrogen the interval between the entropy values should be not less than 0.2 Mn/m^2; for the adiabatic exponent m two values are sufficient in this interval. In addition, along the isobar the temperature interval between the entropy values should be less. As a consequence of the smaller pressure dependence, interpolation for values of m can be carried out for rounded off values of the pressure, which is impossible to do with the entropy.

It is necessary to stress the great simplicity of finding the adiabatic exponent from tabulated data [2]. Since for many gases relatively small temperature drops are observed, then i-s or T-s diagrams cannot in fact give the necessary computational accuracy provided by the data in [2].

It is possible to illustrate the three methods considered for determining the final temperature of an isoentropic process by the following examples. For $p_1 = 4$ Mn/m^2, $T_1 = 300$° K and $p_2 = 10$ Mn/m^2 the value of T_2 for N_2, found from entropy tables, is 392.2° K, from the adiabatic exponent m it is 392.1° K, and from the adiabatic exponent k it is 393.2° K.

The accuracy of Eq. (9) lies within the limits of accuracy of the entropy tables [2] for pressure ratios ≤4. At higher pressure ratios an error begins to appear owing to the averaging of the exponent in (9).

To attain accurate calculations in this case a correction to T_2 can be introduced, equal to the difference between the temperature determined from Eq. (9) and the temperature determined from the entropy s^0 for the ideal gas state. For example, with $p_1 = 1$ Mn/m^2, $T_1 = 300$° K and $p_2 = 10$ Mn/m^2, T_2 for nitrogen, found from the entropy tables is 579.4° K, from the adiabatic exponent m after correction it is 579.4° K, and from the adiabatic exponent k it is 583.6° K .

The isoentropic temperature drop is found using the adiabatic exponent k from the expression

$$\Delta i = z_1RT_1\,\frac{k}{k-1}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k}-1\right]. \qquad (14)$$

Although it may appear to be advisable to use this equation under some conditions [5], its accuracy is generally inferior to the accuracy of determining the temperature drop from the enthalpy. The use of mean heat capacities and Joule-Thompson effect data to calculate the temperature drop [6] is laborious and cannot provide the accuracy obtained using enthalpy tables.

Thus for some important applications, such as the thermodynamic calculation of a gas turbine assembly closed circuit, it is advisable to use the adiabatic exponent m for finding the final temperature of an isoentropic process in a real gas, and to determine the temperature drop from the given initial state and the found final state using the enthalpy in the usual way.

NOTATION

p is the pressure; v is the specific volume; k is an adiabatic exponent; R is the gas constant; T is the absolute temperature; z is the compressibility factor; c_p is the heat capacity at constant pressure; c_v is the heat capacity at constant volume; ΔT is the interval between values of the functions z_{-2}, z_{-1}, z_0, z_1, z_2; m is an adiabatic exponent; s is the specific entropy; Δi is the temperature drop; s^0 is the specific entropy in the ideal gas state. The indices 1 and 2 refer to the start and finish of the isoentropic process, respectively.

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